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The Structure-Immittance Approach for Passive Vibration Control

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Abstract

This paper proposes a new method, the structure-immittance approach, for designing the passive vibration absorbers consisting of inerters, dampers and springs. When considering possible configurations for these elements broadly, one of two exist approaches may be taken, either a structure-based or an immittance-based approach. Both methods have their advantages and disadvantages. In this paper, the new approach combines the advantages of the existing ones. It can both consider a full set of absorber layouts together (the advantage of immittance-based approach), and restrict the complexity, topology and element values of the candidate layouts (the advantage of structure-based approach). The structural immittances covering a full set of possible networks with one damper, one inerter and at most one spring are derived and applied to a civil engineering study. This demonstrates the advantages of the new methodology in being able to identify the optimum configurations for different element constraints.

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Keywords: passive vibration control, inerter, restricted complexity, systematic approach

1. Introduction

Passive vibration control is commonly used in mitigating unwanted vibration in structures, among which one of the most effective tools is the dynamic absorber. To design the passive control device, firstly network layouts containing springs, dampers and masses are presented as candidate control systems and then analysed to identify the optimal configurations. This approach is termed as *structure-based* approach in the present paper. Tuned mass dampers (TMDs) proposed by Frahm [1] as a vibration absorber has been widely used in building structures (e.g. Taibei 101 building) and so on.

Recently, a new mechanical device termed the inerter was introduced by Smith [2] in 2002. It has the property that the generated force is proportional to the relative acceleration across its two terminals. By employing or supplement the mass element with inerters, vibration absorber layouts for building suspensions, such as the tuned inerter damper (TID) [3], tuned viscous mass damper (TVMD) [4] and tuned mass-damper-inerter (TMDI) [5] have been identified. However, among these different suspension layouts, it is not clear which one should be used. It can be easily seen that all the layouts proposed by the structure-based approach have fixed network structures, which restricts the use of the passive impedances and limits the achievable performance of the mechanical system.

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To avoid such problems, the *immittance-based* approach can be used. With this approach, one can obtain an immittance function providing the optimum performance and the corresponding network configuration is synthesized using network synthesis theory (e.g. [6]). Note that the inerter makes this approach possible, prior to its proposal the realisations were very limited. Applications of the immittance-based approach to various mechanical systems have been identified, such as railway vehicles [7], multi-storey buildings [8] and landing gears [9]. With immittance-based approach, although a wide range of possible networks is able to be analysed systematically, the element number and element values can not be predetermined, hence sometimes, networks with unacceptable element values will occur.

The present paper presents a new design method, *structure-immittance* approach, for passive vibration control. We first define a transfer function or functions, that captures all possible arrangements of one damper, one inerter and at most one spring. This transfer function can then be used in an optimisation as is currently done in immittance-based approach. However, the difference is that we can relate the optimised values back to a networks with the set number of elements and we can place limits on component values if it is needed.

This paper is structured as follows. In Section 2, formulation of the structural immittances realisable by a full set of networks with one damper, one inerter and at most one spring is proposed. Application of the structure-immittance approach to a civil engineering structure is illustrated in Section 3, which illustrate the potential advantages of the proposed approach. Conclusions are drawn in Section 4.

2. Structural immittances formulation

We consider a one-port series-parallel network in mechanical domain consisting of springs, dampers and inerters. The structural immittances, that capture all possible arrangements of a set number of elements are defined in this section. Considering the cost and space limits in the application, the number of damper and inerter is restricted to be one and that of springs is up to one. Two cases with zero spring and one spring are analysed in the following.

(*i*) For the case with one damper, one inerter and zero spring, just two networks can be obtained by connecting damper in series or in parallel with the inerter. This can be represented by the *generic network* shown in Figure 1 with the condition that either $c_1 = 0$ and $0 < c_2 < \infty$ or $0 < c_1 < \infty$ and $c_2 = \infty$ to cover the two possible layouts. Note the constructed network is not unique. For example, we could also have c_1 in parallel connection with a series connecting b and c_2 or a network with one damper and two inerters.

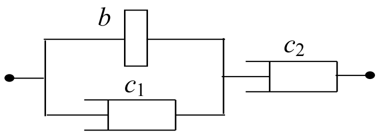

Fig. 1. The generic network for one damper and one inerter case.

The transfer function from force to velocity of the constructed network can be expressed as

$$\frac{F}{v} = c_2(bs + c_1)/(bs + c_1 + c_2) \tag{1}$$

with the condition that one of the parameters c_1 or $1/c_2$ is positive and the other one equals zero. The two possible networks can be analysed using (1) with the constraint on c_1 and $1/c_2$.

(*ii*) For the case with one damper, one inerter and one spring, 8 networks in total can be obtained and the process to formulate the structural immittances is not that straightforward, where four steps are needed. At the first step, two generic sub-networks shown in Figure 2 are constructed, one is inerter-based and the other is damper-based, satisfying the condition that at most one of $1/k_1,\ k_2$ is positive and the others equal zero.

Step 2 is the series and parallel connection of these two networks, hence two networks can be obtained and the one with series connection is shown in Figure 3-step 2. For this obtained network, at most one of the parameters $1/k_1$, k_2, $1/k_3$ and k_4 is positive and the others are all zero. Next, at step 3, we check the redundancy of the springs in the network obtained in step 2. The springs should be checked one by one. Firstly, assuming $0 < k_1 < \infty$ and $k_2 = 0,\ k_3 = \infty,\ k_4 = 0$, a network with inerter, damper and spring in series connection can be obtained, this can also be realised by the condition $k_1 = \infty,\ k_2 = 0,\ 0 < k_3 < \infty$ and $k_4 = 0$. Hence, k_1 is redundant and a modified network

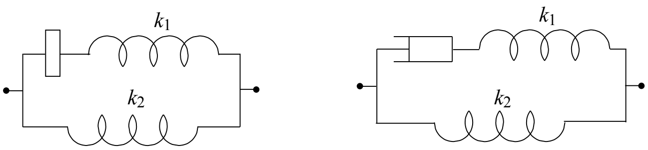

Fig. 2. The networks obtained at steps 1 for one damper, one inerter and one spring case.

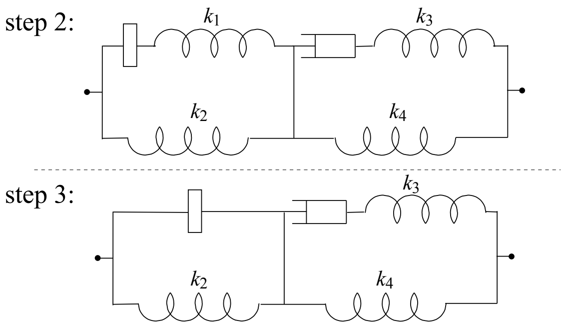

Fig. 3. The networks obtained at steps 2 and 3 for one damper, one inerter and one spring case.

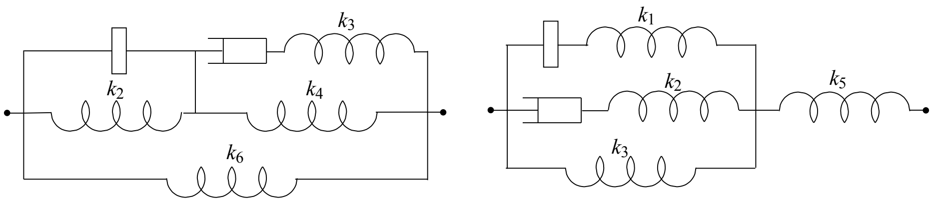

Fig. 4. The networks obtained at steps 4 for one damper, one inerter and one spring case.

structure with $k_1 = \infty$ can be obtained. Move on to spring k_2 in the modified structure, the network obtained with $0 < k_2 < \infty$, $k_3 = \infty$, $k_4 = 0$ is not equivalent to any network obtained with one of the springs k_3, k_4 is positive and finite, so is not redundant. Then considering the k_3 and the k_4, it can be seen that these two can not be removed either. As a result, this network can be simplified to the network of Figure 3-step 3, with the condition that at least two of the parameters k_2, $1/k_3$ and k_4 are equal to zero.

The last step, step 4 involves connecting springs in series and in parallel with the network of Figure 3-step 3. At the beginning, consider adding a spring k_5 in series, it can be checked that the network obtained with $0 < k_5 < \infty$, $k_2 = 0$, $k_3 = \infty$, $k_4 = 0$ is equivalent to that with $k_2 = 0$, $0 < k_3 < \infty$, $k_4 = 0$, $k_5 = \infty$, hence the new spring k_5 is redundant. Instead, consider a spring k_6 added in parallel, this time the network obtained with $0 < k_6 < \infty$ can not be realised by any other springs. Thus the parallel spring k_6 is needed. Any additional spring added in series or in parallel is redundant because those can be covered by either k_3 or k_6 being positive and finite, respectively. As a result, the left-hand network of Figure 4 can be obtained, which requires at least three of the parameters k_2, $1/k_3$, k_4 and k_6 to equal zero.

Following the similar procedure, the other network obtained in step 2 by connecting the two networks shown in Figure 2 in parallel can be formulated into the right-hand network shown in Figure 4, which satisfies at least three of the parameters $1/k_1$, $1/k_2$, k_3, and $1/k_5$ are all zero.

By expressing the force-velocity transfer functions of the two networks shown in Figure 4 and making use of the condition that only one stiffness is positive and finite, we can obtain the following:
Between the two force-velocity structural immittances:

$$Y_1(s) = \frac{bcs^2 + b(k_4 + k_6)s + c(k_2 + k_6)}{bc(1/k_3)s^3 + bs^2 + cs + k_2 + k_4}, \tag{2}$$

$$Y_2(s) = \frac{bc(1/k_1 + 1/k_2)s^3 + bs^2 + cs + k_3}{b(1/k_1 + 1/k_5)s^3 + c(1/k_2 + 1/k_5)s^2 + s}. \tag{3}$$

where $b \geq 0$ and $c \geq 0$, the full set of networks with at most one damper, one inerter and one spring can be realised. And for the function (2), at least three of the parameters $k_2,\ 1/k_3,\ k_4,\ k_6$ must equal zero, whereas for (3), at least three of the parameters $1/k_1,\ 1/k_2,\ k_3,\ 1/k_5$ must equal zero.

3. Application to building vibration suppression

In this section, the structural immittances obtained in the last section are applied to a civil engineering study. A three-storey building model shown in Figure 5 is considered, with floor masses m and the inter-storey elasticity k. The structural damping is taken to be zero as is typically smaller than the control device introduced. Here, a passive mechanical suppression system is assumed to be mounted between the ground and the first floor, with the transfer function $Y(s) = F/v$ representing its mechanical admittance and F is the force exerted by the control device, v is the velocity between the two terminals. The building parameters used in [3,8], namely $m = 1000$ kg and $k = 1500$ kN/m are adopted in this paper, so that results can be compared to the previous work.

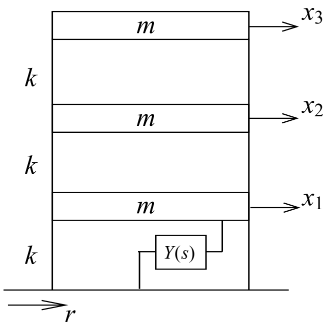

Fig. 5. The three story building model with a vibration suppression device mounted at the bottom.

For choosing the performance criteria, there is a wide variety of cost functions could be studied [10], but here it is the approach we take to address the optimisation problem that is important rather than the detailed cost function dependent results. Hence, the displacements of the building storeys relative to that of the base are considered as the performance index. The objective function is defined as

$$J_\infty = \max\left(\left\|T_{R\to Z_i}(j\omega)\right\|_\infty\right),\ i = 1, \cdots, n \tag{4}$$

where $T_{R\to Z_i}$ denotes the transfer function from R to Z_i, $\left\|T_{R\to Z_i}(j\omega)\right\|_\infty$ is the standard H_∞-norm, which represents the maximum magnitude of $T_{R\to Z_i}$ across all frequencies.

We take the suppression system $Y(s)$ as a inerter-based vibration device including one inerter and one damper considering the cost and space limits in the implementation. The range of the inerter is set to be from 0 kg to 2000 kg and the damping value is chosen as $c \in [10$ Ns/m, 12 KNs/m$]$. In order to obtain the optimal structure with a specific value of b and c, we optimise the objective function J_∞ with the functions ((1)-(3)), making use of patternsearch and fminsearch in Matlab. At each optimisation, the values of b and c are fixed and the parameters need to be optimised are the stiffness shown in these functions. Taken structural immittance (2) as an example, four parameters need to be optimised, namely $k_2,\ k_3,\ k_4$ and k_6, however at any time three of them will be zero due to the constraints, simplifying the optimisation space considerably. Hence, the optimisation can provide accurate results including the optimal configuration and the corresponding value of J_∞. The optimal structures and the corresponding optimal results for the zero and one spring case with respect to the value of the inerter and that of the damper have been shown in Figures 6 and 7, respectively.

From Figure 6(a), it can be seen that for the case where the suppression device has a damper and inerter but no springs, the two possible structures I_1 and I_2, shown in Figure 6(c), both have the corresponding optimal regions versus inertance and damping values. The optimal values of the cost function J_∞ (4) have also been shown in Figure 6(b), where the minimum value of J_∞ is shown as the asterisk and it is obtained as $J_\infty = 13.07$ with $b = 849.9$ kg and

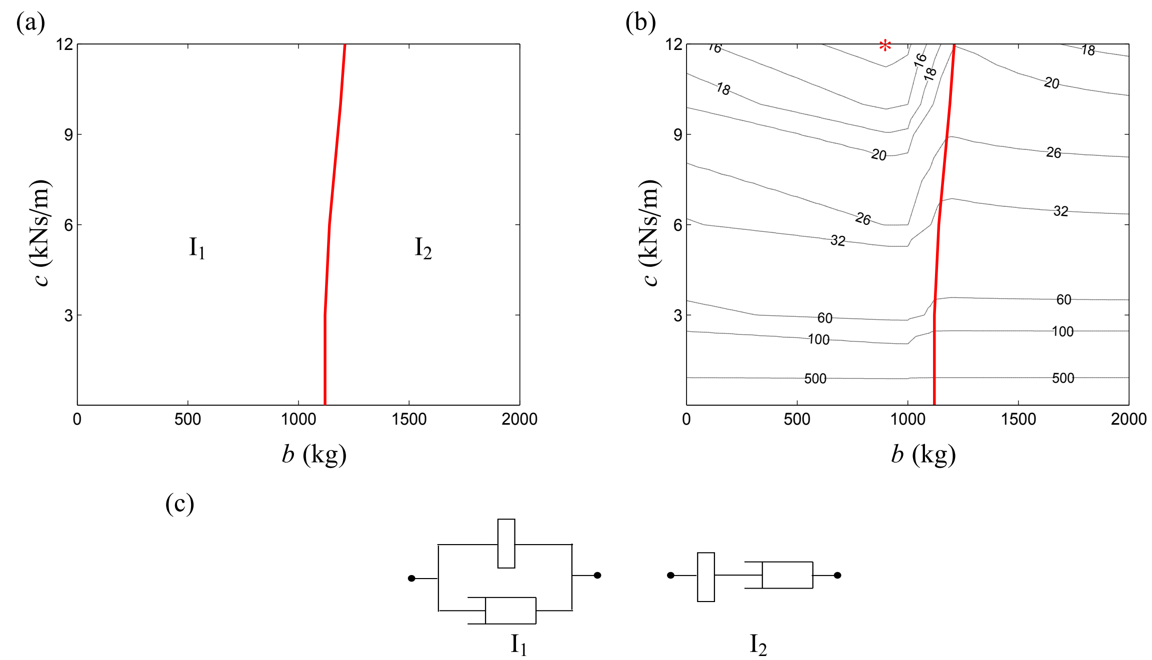

Fig. 6. Optimal results for the one damper and one inerter case: (a) optimal structures with different inertance and damping value, (b) the corresponding values of J_∞, (c) the corresponding optimum structures

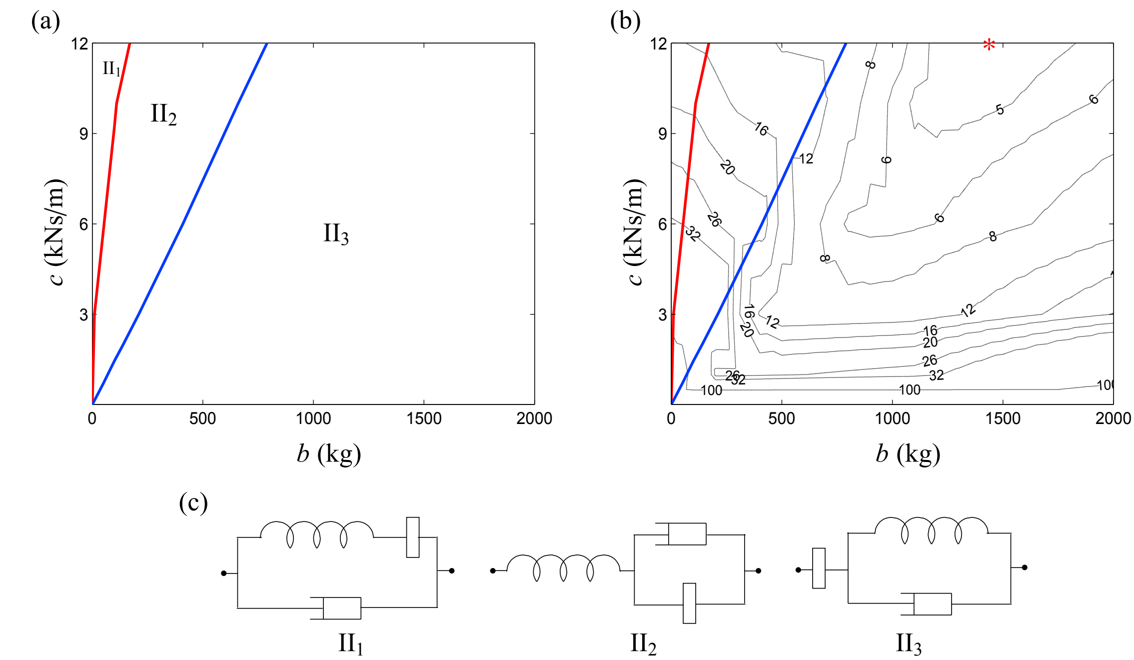

Fig. 7. Optimal results for the one damper, one inerter and one spring case: (a) optimal structures with different inertance and damping value, (b) the corresponding values of J_∞, (c) the corresponding optimum structures.

$c = 12$ kNs/m. It can also be seen that increasing the inertance does not always obtain a better performance, for example, when $c = 9$ kNs/m, $b = 550$ kg, the value of J_∞ is about 20, however, if b is changing to 1250 kg, J_∞ increases to approximately 26, almost 30% larger. The sensitivity of the optimal configuration can also be noticed from Figure 6(b), where in the range of $b \in [900 \text{ kg}, \ 1200 \text{ kg}]$ and $c \in [9.5 \text{ KNs/m}, \ 12 \text{ kNs/m}]$, the performance of the optimum structure I_1 is sensitive to the change of the values of b and c.

For the case where the suppression device includes one spring, out of the 8 possible layouts, 3 networks shown in Figure 7(c) provide the best performance across the different b, c region, see Figure 7(a). The contour plots for the optimal values of the objective function J_∞ (4) are shown in Figure 7(b), and the asterisk in the figure represents the minimum value of the cost function with $J_\infty = 4.45$ when $b = 1450.3$ kg and $c = 12$ kNs/m. It can be calculated that the minimum value of the objective function improves about 65.95%, comparing with the previous case with no springs. From Figure 7(b), we can see the structure II_3 is very sensitive to the change of the parameters b and c in some regions, especially $b \in [200\text{ kg}, 1000\text{ kg}]$ and $c \in [1\text{ kNs/m}, 3\text{ kNs/m}]$. From the results shown in Figure 6(b) and 7(b), we note that with the same b and c, the suppression device with an additional spring can always provide a much better performance. Besides, if we want to get an acceptable value of J_∞, say $J_\infty = 16$ with no spring structure I_1 could be chosen with $b = 750$ kg, $c = 10.5$ kNs/m, whereas with one spring, $J_\infty = 16$ can be obtained with a smaller b and c (500 kg and less than 9 kNs/m, respectively).

4. Conclusion

A new approach for the identification of optimum passive vibration control device, namely the structure-immittance approach, has been discussed. A full set of networks consisting of one damper, one inerter and at most one spring was anaylsed systematically, using the formulated structural immittances. With these immittance functions, both the complexity of the layout in terms of the number of elements, and the value of each element can be restricted if desirable, representing a significant advantage over the immittance-based approach. By applying the structure-immittance approach to design vibration suppression device for a three-storey building model, optimal configurations for the zero and one spring case were obtained over a range of inertance and damping values. Furthermore, the approach indicates the sensitivity of the device. These can provide guidance for selecting the appropriate suspension device considering the element numbers, the element values and the robustness.

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